Extension of tailored design method: illumination with a collimated annular source

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ABSTRACT

Many commercially available sources can be modeled as collimated, annular sources. We use a tailored central-ray method to concentrate light from such a source onto a cylindrical target. The design method is a straightforward extension of methods previously described for point sources. The device was developed and is now being used for a commercial application.

Keywords: non-imaging optics, illumination, ringlight

1. INTRODUCTION AND BACKGROUND

Classical non-imaging designs have primarily focused on problems where étendue conservation played a critical role in the design challenge. There are many problems, however, in which étendue need not be conserved, and where the primary requirement is to match the flux distribution to a particular pattern. A standard example is Winston's treatment of a point source, where the reflector is "tailored" to direct the outgoing ray fan into a selected angular distribution. (This approach is described in more detail below.)

We want to discuss a modest extension of this approach for use with a collimated light source (equivalent to a source at infinite distance), and particularly with an annular source. This model represents, for example, of the output of many standard reflectorized incandescent lamps and LED ring-lights. We have developed a secondary reflector which couples such a source to a cylindrical target, with good control over the light distribution on the target. This device was designed, simulated, built, and is now being used in a commercial application.

1.1. Review of Winston's point-source approach

Since our method is closely related to Roland Winston's derivation of distribution-tailoring reflectors for a point source[1], we will start by reviewing his approach. The geometry is shown in Figure 1. There are two basic equations: (1) a "flux distribution relation" which contains the mapping of source distribution coordinates onto target distribution coordinates needed to obtain a given output flux distribution; and (2) the law of reflection, given as a differential equation for the reflector surface, expressed in the same coordinates as the flux distribution relation.

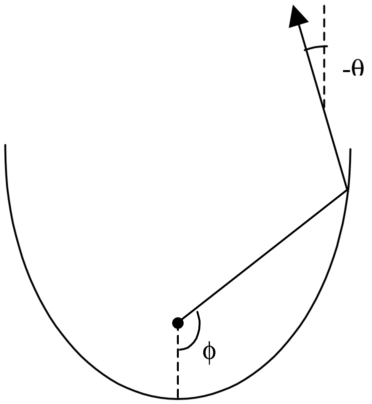

Figure 1. Coordinate system for point-source example

1.1.1. Flux distribution relation

The flux distribution relation is obtained from the source and desired target power distributions:

$$\begin{aligned} dP_{source} &= P'_{source}(\phi)\cdot 2\pi\ d\cos\phi \\ dP_{target} &= P'_{target}(\theta)\cdot 2\pi\ d\cos\theta \end{aligned} \qquad (1)$$

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where $P'_{source}(\phi) = \dfrac{dP(\Omega_{source})}{d\Omega_{source}}$ expressed as a function of ϕ, etc. If the source is uniform and isotropic and we collect it over a total angle ϕ= 0 to ϕ_{rim}, and if we want the output distribution to be uniform within $\pm\theta_{out}$, then the normalized power distributions are

$$\begin{aligned} dP_{source} &= \frac{P_0}{2\pi(1-\cos\phi_{rim})}\cdot 2\pi\ d\cos\phi = \frac{P_0}{1-\cos\phi_{rim}}d\cos\phi \\ dP_{target} &= \frac{P_0}{2\pi(1-\cos\theta_{out})}\cdot 2\pi\ d\cos\phi = \frac{P_0}{1-\cos\theta_{out}}d\cos\theta \end{aligned} \tag{2}$$

where P_0 is the total power collected by the reflector. Setting dP_{source} = dP_{target} and solving the resulting equation, with the boundary conditions θ=0 at ϕ=0 and $\theta=\theta_0$ at $\phi=\phi_0$ gives the flux distribution relation between ϕ and θ:

$$\cos\theta = \frac{1-\cos\theta_{out}}{1-\cos\phi_{rim}}\cos\phi + \frac{\cos\theta_{out}-\cos\phi_{rim}}{1-\cos\phi_{rim}} \tag{3}$$

1.1.2. Law of reflection

The law of reflection, expressed in polar coordinates, is

$$\frac{d\ln R(\phi)}{d\phi} = \tan\left(\frac{\phi-\theta}{2}\right) \tag{4}$$

1.1.3. Results

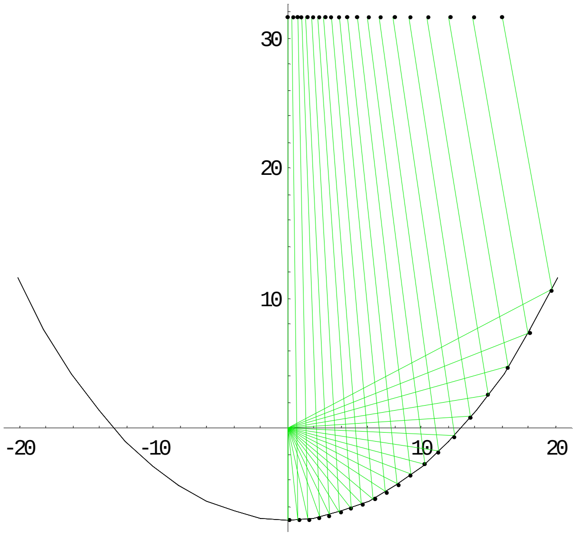

Figure 2. Point-source example with θ_{out}=10° and ϕ_{rim}=120°.

Combining these equations gives a differential equation for the reflector. Figure 2 shows a ray-traced example. The reflector resembles a paraboloid, but the slope is tailored just enough to spread the output through a uniform fan of ±10°. Only one side of the reflector is shown ray-traced, and it fills 0 to -10°; the other side fills 0 to10°.

2. EXTENSION OF METHOD TO COLLIMATED LIGHT: TAILORED REFLECTOR FOR ILLUMINATING A CYLINDER WITH AN ANNULAR SOURCE

For a recent application, we were presented with the problem of evenly illuminating a cylindrical target using a reflectorized incandescent lamp or an LED ring-light. Commercially available sources produced reasonably collimated outputs within fairly uniform annular spatial patterns. The problem was to deliver this output uniformly and efficiently to an axially situated cylinder. The geometry is shown in Figures 3 and 4, with fairly typical dimensions. With collimated sources the source étendue is much less than the target étendue.

Our initial design attempts used edge-ray designs, with restricted target angles to match the 2D étendue. These attempts failed miserably—not only were the reflectors far too large, but the upper edge of the cylinder was illuminated much more intensely than the remainder. We also attempted modified edge-ray designs, but still with little success in achieving uniform illumination.

The ultimately successful design used an extension of the above-described tailoring method. Instead of mapping source angle ϕ to target angle θ, we simply mapped source radial position r_r to the target position z_{cyl}. (Note that since the source is collimated the r coordinate on the source is equivalent to the r coordinate on the reflector.)

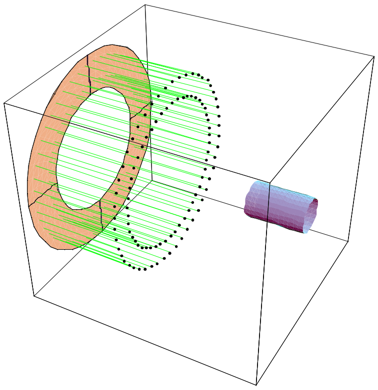

Figure 3. Three-dimensional rendering of source and target

2.1.1. *Flux distribution relation*

For uniform source and target distributions, the normalized flux distributions are

$$\begin{aligned} dP_{source} &= \frac{P_0}{\pi\left[\left(\frac{D_2}{2}\right)^2-\left(\frac{D_1}{2}\right)^2\right]}\cdot 2\pi\, r_r\, dr_r \\ dP_{cyl} &= \frac{P_0}{h}dz_{cyl} \end{aligned} \tag{5}$$

These equations are easily combined and integrated to yield

$$z_{cyl} = \frac{h}{\left(\frac{D_2}{2}\right)^2-\left(\frac{D_1}{2}\right)^2}\left[{r_r}^2-\left(\frac{D_1}{2}\right)^2\right] \tag{6}$$

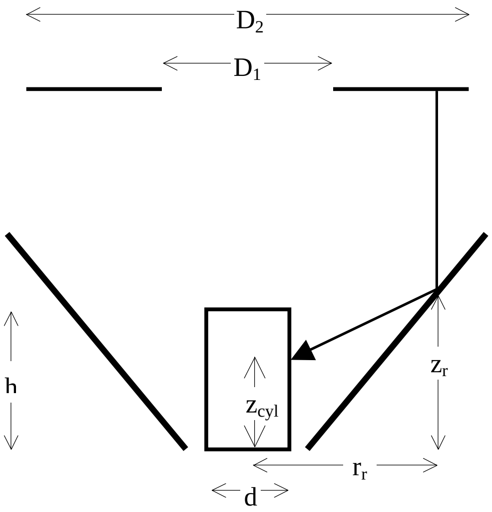

Figure 4. 2D cross-section showing layout and coordinate system for reflector. r_r and z_r are the coordinates of a point on the reflector which reflects a ray to height z_{cyl} on the target. D_1 and D_2 are the inner and outer diameters of the annular source.h and d are the dimensions of the cylinder.

2.1.2. *Law of reflection*

For this problem polar coordinates are clearly inappropriate. Instead it makes more sense to use Cartesian coordinates in the cross-sectional plane. The reflector is defined by the function $z_r(r_r)$ and the slope angle of the reflector is atan($z_r'(r_r)$). Then the law of reflection is

$$\begin{aligned} z_{cyl} &= z_r(r_r)-\left(r_r-\frac{d}{2}\right)\tan\left(2\arctan z_r'(r_r)-\frac{\pi}{2}\right) \\ &= z_r(r_r)-\left(r_r-\frac{d}{2}\right)\frac{z_r'(r_r)^2-1}{2z_r'(r_r)} \end{aligned} \tag{7}$$

2.1.3. *Theoretical results*

Equations 6 and 7 can be combined to produce a differential equation for $z_r(r_r)$. With the addition of a boundary condition specifying the bottom edge of the reflector, $z_r(D_1/2)$, the equation can be numerically integrated. The result is shown and ray-traced in Figure 5. Note that in this 2D cross-section the clustering of rays is higher toward to bottom of the cylinder. This density variation compensates for the fact that, in 3D, the outer portion of the reflector represents significantly more area.

Not surprisingly, full 3D Monte Carlo ray-tracing verifies that with a perfectly collimated and uniform source, this design algorithm provides essentially perfect uniformity and efficiency. With realistic angles, the performance is still excellent. Figure 6 shows results for 2 different angles. The cylinder z-coordinate has been normalized to the total height h, and the intensity has been normalized to the intensity of a perfectly uniform system with 100% geometric efficiency. Performance is only slightly degraded for a source with angle ±9.5°.

2.1.4. Experimental results and discussion

A design according to the approach described above was fabricated and tested. The nature of the application unfortunately made direct, quantitative measurements of the flux distribution impossible. Measurements could have been made by creating a special benchtop system, but unfortunately the scale of the project did not justify this expense. However, the actual function of the device indicated a very uniform flux profile, in agreement with the predictions of Figure 6. The device is currently being sold and functioning well in its intended commercial application.

Further refinement of this algorithm is of course possible. Modifications to Equations 5 can be implemented to provide on-uniform target distributions or to deal with source non-uniformity. Both of these paths were explored by ray-tracing, which verified the flexibility of the approach. Likewise, source angular extent could be included by the methods outlined by various combinations of Winston, Ries, Gordon and Rabl.[3-6] However, the strong performance of the first-order designs made all of these additional steps unnecessary.

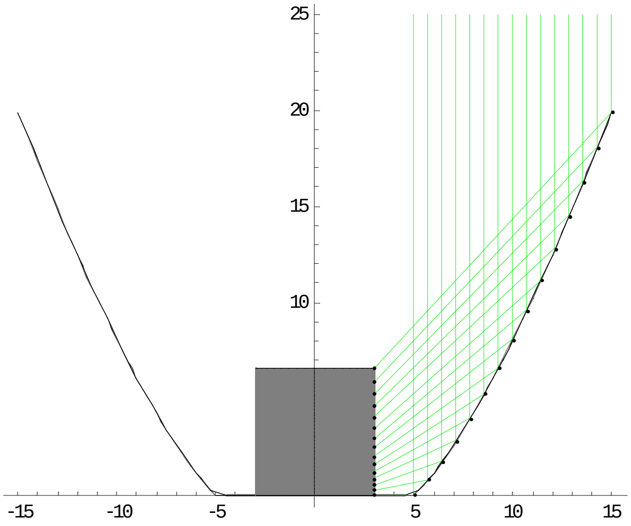

Figure 5. Ray-trace of sample tailored reflector

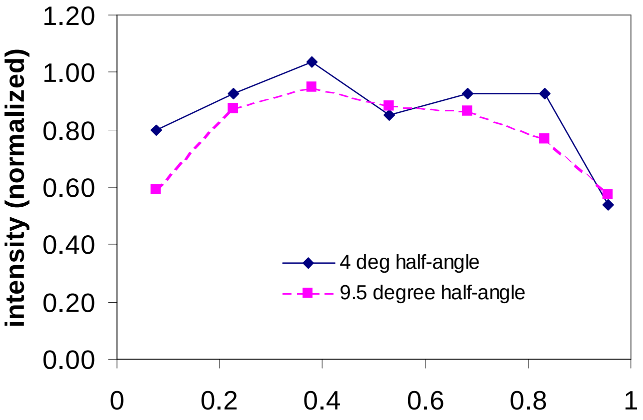

Figure 6. Monte Carlo 3D raytrace results for non-collimated sources

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